invited paper, IEEE Radio and Wireless Simposium RWS 2006, San Diego, California, USA, January 17-19, 2006

TH4B-1

Small World routing: A methodological framework for driving the emerging topology of energy-constrained multi-hop wireless networks

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***Abstract* — A Small World topology may result in particularly appealing features for wireless ad-hoc and sensor networks, thanks to its combination of high local connectivity and of short distance between terminals. In this work we propose a routing strategy aiming at driving the network topology of energy-constrained ad-hoc networks towards Small Worlds by combining short and long distance links in the end-to-end paths. Results of simulations show that the proposed strategy does lead to the desired topology, and allows excellent network performance in comparison with strategies leading to different network topologies.**

***Index Terms* — Small World, Routing, UWB, Cognitive radio.**

I. Introduction

Small World networks were introduced in the 60's by Milgram [1] for modeling social networks. A quite well-known finding of Milgram's work shows that the average distance between two individuals in U.S. population, defined as the number of intermediary individuals required to exchange information, is about 6, independently of the size of the network. This concept is generally referred to as the "six degrees of separation" feature. Small World networks were fully analytically described in the 70's in [2], with further refinement in the 90's [3]-[4], leading to the definition of the Small World pattern as a strong local connectivity combined with a short average distance between any pair of individuals in the network.

Moving to communication networks, the research carried out on wired networks highlighted that the Small World behavior can be observed in large-scale data networks, such as peer-to-peer networks and internet [5]-[6]. A Small World-like topology is particularly appealing in the case of wireless communication networks, where topology may vary in time according to both living status of the nodes and routing metrics ruling the path selection algorithms used to convey packets from source to destination. In this framework, the combination of high local connectivity and short average distance characterizing a Small World-like topology may provide high robustness to link failure, while guaranteeing low delays. Some research has been performed on the application of the Small World concept to wireless networks. Previous work mainly focused either on replicating the work of Watts and Strogatz in a wireless context [7], by analyzing the impact of shortcuts on topology characteristics, or on applying the Small World concept to network planning in infrastructure-based wireless networks, such as cellular or hybrid networks [8]. In [8], the authors propose a strategy for placing Fixed Relay Nodes (FRNs) in the vicinity of cellular Base Stations, and analyze the impact of such a strategy on the topology of potential links between two terminals, defined as the physical visibility between the two terminals.

Oppositely, in this work we adopt the Small World concept as a driving paradigm in shaping a *logical* topology of the network: in our network graph, an edge between two nodes is present when the radio link between the corresponding network terminals is not only potential but also in current use. The distinction with the topology definition of [8] can be highlighted by thinking of a network of terminals in full coverage with one another: that would be fully connected for [8] while any topology could emerge in our approach.

We focus our analysis on multi-hop wireless networks with energy constraints. Ad-hoc and sensor networks are typical examples of this class of networks, where stringent requirements on energy efficiency are imposed for at least two reasons: 1) for increasing network lifetime, and 2) for improving coexistence with other radio systems sharing the same spectrum.

Within this framework Ultra Wide Band (UWB) sensor networks represent a role example of systems characterized by limited allowed power [9]. UWB signals spread over very large bandwidths. As a consequence, regulatory bodies impose severe limitations on UWB power density in order to allow the coexistence of UWB with narrow-band systems. In the view of improving such coexistence, the concept of cognitive UWB networks, that adapt their behavior to the actual status of both UWB and co-located narrow-band systems, is now actively investigated [10]. In this framework, the problem of limiting the cumulative network generated power rather than the power emitted by a single device as set by emission masks is particularly relevant. The definition of routing strategies capable of adapting the logical network topology so to optimize the use of available power as a function of external conditions is of prime importance.

A first step in this direction was the definition of a method for setting up connections by optimizing a power-dependent cost function in UWB networks [11-12]. Results showed that the adoption of multi-hop communication paths between terminals physical visibility increases network lifetime and reduces emitted power [12]. Since emitted power is the main factor driving the selection of end-to-end paths, the strategy proposed in [12] strongly favors short distance links, leading to a topology with strong local connectivity but high average distance between terminals, with a negative impact on end-to-end delay. In this context, a routing protocol capable of driving the network to a Small World-like topology could improve the end-to-end delay performance thanks to the low average distance, while preserving the strong local connectivity and thus a high energy efficiency.

Moving from the above considerations, in this work we propose a routing strategy capable of driving an energy-constrained ad-hoc network to a Small World-like topology. The proposed strategy is compared with two strategies minimizing power and number of hops, in terms of both network performance and topology of the emerging network of active links.

The paper is organized as follows: Section II describes the key features of a Small World topology, while Section III defines the system model and introduces the different routing strategies. Section IV provides the simulation settings and analyzes the simulation results. Section V draws conclusions.

II. The Small World Concept

The topological characteristics of a network, as represented by a connected graph composed of n nodes, are usually described by means of two topology-related parameters [3]:

- The characteristic path length L;
- The clustering coefficient or *cliquishness* γ.

The *path length* L is defined as the distance between any pair of nodes (i, j), defined as the minimum number of edges connecting i to j, averaged over all possible pairs.

The definition of *cliquishness* γ is based on the concept of *neighborhood*. The neighborhood $\Gamma(v)$ of a node v is the subgraph composed of all the k_v nodes adjacent to v, i.e. at distance 1 from v. If we indicate the number of edges of the neighborhood as $\left|E(\Gamma(v))\right|$, the clustering coefficient for node v is defined as:

$$\gamma_v = \left|E(\Gamma(v))\right| \Big/ \binom{k_v}{2} \qquad (1)$$

The clustering coefficient for the whole graph, or cliquishness, γ is defined as the average of γ_v over all nodes v in the graph:

$$\gamma = \frac{1}{N}\sum_{v=1}^{N} \gamma_v \qquad (2)$$

Different networks with the same number of nodes and edges can be characterized by highly different values of L and γ. Two extreme cases are the following:

- **Regular networks** – characterized by a local organization pattern which is repeated all over the network, they show a high value of path length ($L \propto n$) and a high value of γ.
- **Random networks** – characterized by random connections between nodes without regular pattern, they show a low value of L ($L \propto \log(n)$) and a low value of γ.

Watts and Strogatz studied the transition region between regular and random networks and identified a class of networks characterized by a graph showing a low value of the path length L combined with a high value of the cliquishness γ: the so-called **Small World networks**.

As already noted, a Small World-like topology is an appealing solution in the design of an efficient wireless network, since it combines the redundancy at the local level of a regular network with the scalability of a random network. In the next section we will define a routing algorithm that aims at driving the topology of the network towards a Small World topology.

III. System Model and Routing Strategies

In this work we consider an ad-hoc network of N terminals in fixed positions. The terminals are in full visibility, so that a direct link is possible between each pair of terminals in the network. Each terminal is characterized by a maximum transmission rate B_{MAX}. We assume furthermore that the whole network is subject a to an energy constraint that imposes an upper limit to the global emitted power. In our model, the energy constraints imposed to the network are taken into account by introducing a Network Cost Function (NCF), defined as the sum of the cost of all active connections.

The cost of each connection is evaluated by defining an additive link cost function. For a generic pair *(i,k)* of terminals, the link cost function *LC(i,k)* is defined as follows:

$$LC(i,k) = C_0 \cdot [d(i,k)]^2 + C_1 \cdot \frac{B(i,k)}{100}[d(i,k)]^2 \qquad (3)$$

where *d(i,k)* is the physical distance between the two nodes and *B(i,k)* is the normalized rate required for the link. The first term in (3) takes into account the overhead required for setting-up and maintaining the link, while the second term models the power emitted for transmitting at a rate *B* over a distance *d*.

The cost of a multihop connection between two terminals *i, j* involving *n* intermediate terminals $k_1 \dots k_n$ is then given by the sum of the costs of all composing links, and indicated as $C(i,k_1,\dots,k_n,j)$. Since the cost of each link is evaluated by means of a power-related function, the NCF represents the overall emitted power in the network; we can thus introduce the network-wide energy constraint by imposing a maximum value NCF_{MAX} to the NCF.

In the above framework we investigated the behavior of three different routing strategies, defined in the following for a connection request between terminals *i* and *j*:

Single Hop – this strategy always selects the direct link between *i* and *j*, if $B_i \geq B(i,j)$;

Multihop – this strategy selects the optimal route that meets the condition $B_i \geq B(i,j)$ by adopting as a distance metric the link cost function in (3). Path selection is performed using the Dijkstra algorithm;

Small world – this strategy selects the shortest route between *i* and *j* that meets the energy constraint imposed on the network. The path selection algorithm is based on the definition of a **cluster** centered on each terminal *k* in the network, defined as a circle of radium *r* shorter than the radio range of the terminal, and indicated in the following as *Cluster(k)*.

1. a) If the link *i - j* is admissible, that is if conditions:

   C1) $NCF + C(i,j) \leq NCF_{MAX}$

   C2) $B_i \geq B(i,j)$

   are both met, the direct link is selected and the procedure ends;

   b) If C2) is not met the connection is refused, and the procedure ends;

   c) otherwise the algorithm proceeds to step 2.

2. For each terminal $k \in Cluster(i)$:

   a) Conditions

   C3) $NCF + C(i,k) \leq NCF_{MAX}$

   C4) $B_k \geq B(i,j)$

   are verified. If either of them is not met, *k* is excluded from the path selection and the next node in the cluster of *i* is considered;

   b) Condition

   C5) $NCF + C(i,k,j) < NCF_{MAX}$

   is verified. If C5) is met, an admissible path was found, and it is stored before moving to the next member of *Cluster(i)*.

After all members of *Cluster(i)* have been considered, the following cases are possible:

- one or more nodes met all conditions, and thus one or more paths in two hops were found: among these, the path at lower cost is selected, and the procedure ends.
- none of the nodes satisfied C3) and C4): in this case no admissible path exists, and the procedure ends.
- one or more nodes satisfy C3) and C4) but not C5): for each of such nodes the algorithm applies recursively steps 2.a), and 2.b). For example, assuming that *k* is one of such nodes, conditions C3), C4) and C5) are checked for each node *l* such that $l \in Cluster(k)$ and $l \notin Cluster(i)$, applying C3) and C4) to the path $i - k - l$, and C5) to the path $i - k - l - j$, respectively.

The procedure goes on until either one admissible path is found, or all nodes fail to satisfy conditions C3) and C4).

Note that in the case of the Small World routing strategy, the selected path always meets the limit set by the NCF_{MAX} value. For the Single Hop and Multihop strategies, the cost C_{CONN} of the selected path is evaluated; if the condition $NCF + C_{CONN} < NCF_{MAX}$ is satisfied, the connection is activated, otherwise it is refused.

IV. Simulation Results

The routing strategies defined in Section III were compared in a scenario consisting in a network of 25 terminals disposed on a ring lattice of radium 10 m. The radium of the cluster was set to $r = 5$ m, and the coefficients of the link cost function were set to $C_0 = 0.7$, $C_1 = 1$. Connection requests were generated following a Poisson distribution with average inter-arrival time $\lambda = 10$ s and average duration $T = 1800$ s.

Figures 1 and 2 show the path length and cliquishness, while Figs. 3 and 4 present the percentage of accepted connections and the average number of hops per connection for the above routing strategies.

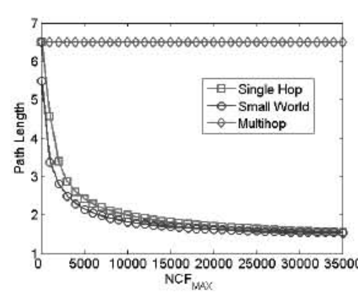

Fig. 1. Path Length as a function of NCF_{MAX} for the Single Hop, Small World and Multihop routing strategies.

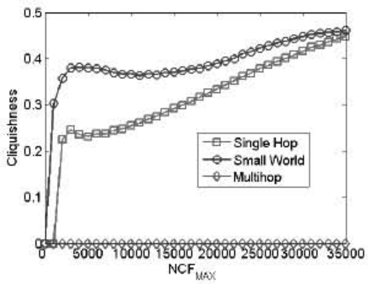

Fig. 2. Cliquishness as a function of NCF_{MAX} for the Single Hop, Small World and Multihop routing strategies.

Figures 1 and 2 highlight that the Small World strategy leads to values of path length and cliquishness typical of a Small World topology, in particular for low NCF_{MAX} values, that are the most relevant in modeling an energy-constrained network. The value of the cliquishness for the Multihop strategy, equal to 0, is coherent with the choice of the cluster radium r, that leads for each node k to a subgraph composed of $k_v = 2$ nodes, and thus $\gamma_v = 0$.

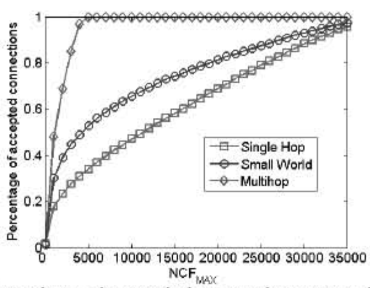

Fig. 3. Percentage of accepted connections as a function of NCF_{MAX} for the Single Hop, Small World and Multihop routing strategies.

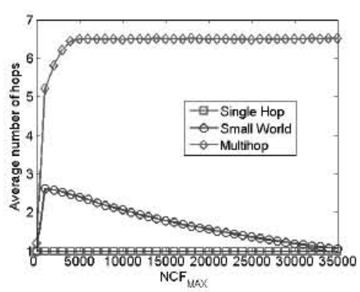

Fig. 4. Average number of hops per connection as a function of NCF_{MAX} for the Single Hop, Small World and Multihop routing strategies.

Figures 3 and 4 show that the Small World strategy leads to a higher percentage of accepted connections compared to the Single Hop, while guaranteeing a lower average number of hops compared to the Multihop strategy.

V. CONCLUSION

In this work the adoption of a routing strategy that leads to a convergence of the topology of an energy-constrained ad-hoc network towards a Small World topology is proposed. Results highlight that the proposed strategy successfully drives the network to a Small World topology, and leads to high network performance in strongly energy-constrained scenarios. The network module should thus take into account network topology in optimizing performance for energy-constrained networks.

ACKNOWLEDGEMENT

This work was partially supported by the European Union under the framework of the HYCON Network of Excellence (contract number FP6-IST- 511368).

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