

# Power limits fulfilment and MUI reduction based on pulse shaping in UWB networks

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**Abstract**—Pulse shaping in Ultra-Wide Band (UWB) networks based on Impulse Radio (IR) is a viable way for adapting the Power Spectral Density (PSD) of transmitted signals to spectral requirements. The proposed pulse shaping method is based on linear combination of a set of base waveforms obtained by differentiation of the Gaussian pulse. Strategies for selecting linear combination coefficients are proposed and approximation of emission masks for UWB indoor systems based on linear combination of the above base functions is analyzed. It is shown that linear combination of base waveforms fulfils spectral requirements with higher efficiency than that achieved by a single waveform. The adoption of pulse shaping in the reduction of Multi-User Interference (MUI) between different UWB networks is finally proposed and evaluated by means of simulations.

**Key words** – Pulse shaping, Ultra-Wide Band, power limits, Multi-User Interference

## I. INTRODUCTION

The FCC regulation [1], released in April 2002, allows for the first time intentional UWB emissions and sets the emission limits for UWB devices for several services, ranging from indoor and outdoor communications to imaging and medical applications. The fulfilment of the above emission limits requires careful design of UWB devices, both in terms of maximum emitted power and shape of the Power Spectral Density (PSD) of the emitted signal. In the case of Impulse Radio UWB (IR-UWB) pulse shaping is a straightforward manner for modifying the PSD of the emitted signal in order to meet the limitations set by FCC emission masks. Pulse shaping enables however innovative solutions for other aspects of UWB networks design, such as Multi-User Interference (MUI) mitigation.

In this work we propose a method to shape the transmitted pulse, based on combination of a set of base waveforms. A Gaussian pulse, which is the most commonly adopted waveform for UWB, is assumed as initial waveform. Two different ways of modifying the waveform are then analyzed: pulse width variation and differentiation. The effect of these techniques on PSD is analyzed, and a set of base functions composed by the derivatives of the Gaussian pulse is proposed. The efficiency of linear combination of the base functions in the approximation of FCC emission mask is then evaluated. Finally, the adoption of different pulse shapes for the purpose of mitigating Multi-User Interference is analyzed, and simulative results are presented.

The paper is organized as follows. Section II defines the UWB signal and describes the emission limits set by FCC masks. Section III describes the properties of Gaussian waveforms and analyzes the effect of pulse width variation and differentiation on PSD. Section IV deals with approximation of emission masks obtained by combining a set of derivatives of the Gaussian pulse. Section V analyzes the effect of pulse shaping on MUI. Section VI presents the conclusions.

## II. UWB DEFINITION AND REGULATION

The FCC regulation [1] indicates that any signal with either fractional bandwidth greater than 0.2 or bandwidth higher than 500 MHz falls into the UWB category. We consider here the most common version of UWB based on the transmission of very short (picosecond) pulses emitted in periodic sequences, in an Impulse Radio (IR) fashion. In order to increase robustness of transmission and control single pulse energy,  $N_s$  pulses are used for each transmitted symbol. Modulation is binary PPM. The transmitted signal is expressed by:

$$s(t) = \sum_{i=-\infty}^{\infty} \sum_{j=0}^{N_s-1} p(t - jT_s - b_i\tau) \quad (1)$$

where  $p(t)$  is the pulse,  $T_s$  the basic time interval between two consecutive pulses, and  $T_b = N_s * T_s$  is the bit duration. Information bits are coded in the sequence of  $b_i$ 's. Multiple access is achieved by using time-hopping codes and, for multi-user communication with  $N_u$  users, the received signal writes:

$$s_{rec}(t) = \sum_{k=1}^{N_u} \sum_{i=-\infty}^{\infty} \sum_{j=0}^{N_s-1} p(t - jT_s - c_j^{(k)}T_c - b_i^{(k)}\tau) \quad (2)$$

where index  $k$  refers to user  $k$ ,  $1/T_c$  is the chip rate, and  $c_j$  is an element of the code word with  $0 \leq c_j \leq N_h$  and  $N_h * T_c < T_s$ . (2) shows that the time-hopping code provides an additional shift of  $c_j * T_c$ .

The FCC limits apply to the transmitted signal  $s(t)$  for a single user. Such limits are set by means of emission masks which set an upper bound for the PSD of emitted signal. Different limits are set for different UWB-based services. Table 1 presents the limits for indoor systems.

TABLE I. FCC EMISSION LIMITS FOR UWB INDOOR SYSTEMS

Frequency range (GHz)	<0.96	0.96–1.61	1.61–1.99	1.99–3.1	3.1–10.6	> 10.6
UWB EIRP (dBm)	-41.3	-75.3	-53.3	-51.3	-41.3	-51.3

The choice of the impulse response  $p(t)$  in (1) is crucial in fulfilling the emission mask, since the pulse shape determines the PSD of the transmitted signal, as it will be shown in the next section.

### III. PULSE SHAPE IN IMPULSE RADIO

Several pulse shapes have been proposed such as the Laplacian [2], compositions of Gaussian pulses having same length and reversed amplitudes with a fixed time gap between the pulses [3], or Hermite pulses [4]. Recently, a pulse shaping technique based on prolate spheroidal wave functions was proposed for generating pulse shapes which fit the FCC emission masks [5]. The most commonly adopted shape for the pulse in UWB is however modelled as the second derivative of the Gaussian pulse, as proposed by Win and Scholtz [6]. This pulse is often referred to as the pulse at the receiver i.e. after passing through the transmitter and receiver antennas. Since we are here interested in the generated pulse at the transmitter, however, we will start our analysis by considering the Gaussian pulse. Typical time domain representation for the Gaussian pulse is:

$$p_G(t) = \pm \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\left(\frac{t}{2\sigma^2}\right)} = \pm \frac{\sqrt{2}}{\alpha} e^{-\frac{2\pi^2}{\alpha^2}} \quad (3)$$

where  $\alpha=4\pi\sigma^2$  is the shape factor and  $\sigma^2$  is the variance of the Gaussian.

We further consider amplitude-normalized waveforms in order to be able to compare in a straightforward manner waveforms corresponding for example to different derivatives, which are characterized by different amplitudes. Since in this section we consider a single pulse, which is an energy signal, we will analyze the Energy Spectral Density (ESD) of such signal. The ESD of a waveform can be expressed to a first approximation by:

$$ESD(f) = |P(f)|^2 = \left| \int_{-\infty}^{+\infty} p(t) \cdot e^{-j2\pi ft} df \right|^2 \quad (4)$$

where  $p(t)$  represents the single pulse. The Gaussian waveform, and the corresponding ESD, are shown in Fig. 1 and Fig. 2 respectively, for  $\alpha=0.714$  ns.

The Gaussian pulse is well suited for the pulse shaping operations we are interested in, since its shape can be modified in a straightforward way by acting on the shape factor  $\alpha$ , and infinite new waveforms can be obtained by differentiating the original pulse. Properties of Gaussian derivatives are also investigated in [7], where a method for selecting a single derivative of the Gaussian pulse which fits the FCC masks is proposed.

In the following, we analyze in detail the effect of pulse width variation and differentiation on pulse shape and corresponding ESD characteristics.

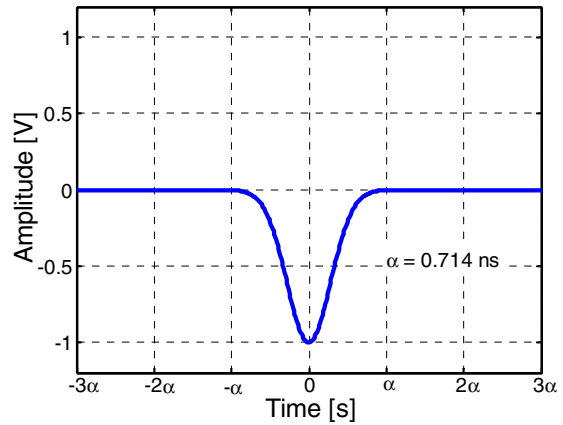


Figure 1. Gaussian pulse

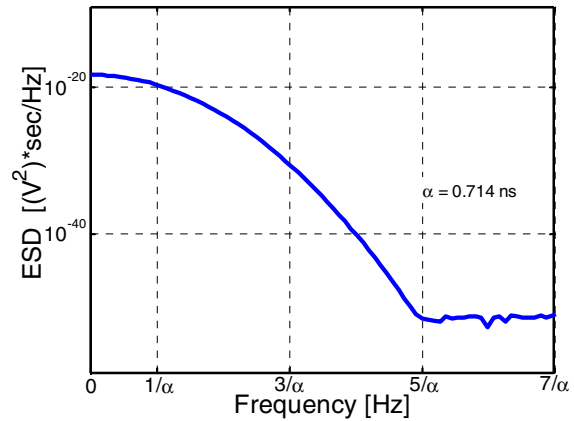


Figure 2. Energy Spectral Density of Gaussian pulse

The pulse width is tightly related to the shape factor  $\alpha$ . Reducing the value of  $\alpha$  shortens the pulse, and thus enlarges the bandwidth of the transmitted signal. This effect is presented in Figs. 3 and 4, which show the variation of pulse duration and ESD with  $\alpha$  varying from 0.414 to 1.014 ns.

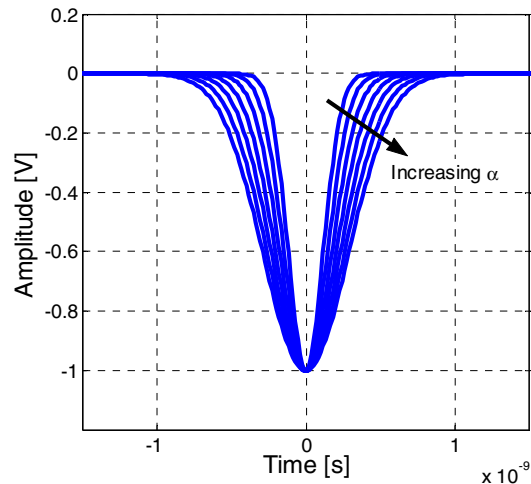


Figure 3. Effect of  $\alpha$  variations on pulse width

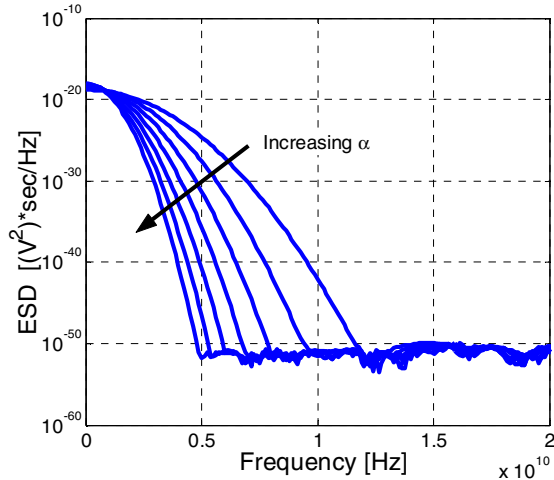


Figure 4. Effect of  $\alpha$  variations on Energy Spectral Density

Note that the Gaussian pulse has infinite duration leading to unavoidable overlap between pulses and intersymbol interference. It is reasonable however to consider the Gaussian pulse limited in its duration to  $T_m$  as defined by limiting the cut-out energy below a given threshold. Under this assumption an upper limit for  $\alpha$  is given by pulse duration  $T_m$  which cannot exceed the chip duration  $T_c$ , while a lower limit is given by technological limitations in generating extremely short pulses.

As regards pulse differentiation, as already noted, the Gaussian pulse can be derived infinite times. In the following we focus the analysis on the first 15 derivatives of the Gaussian pulse. The differentiation has an effect on the peak frequency  $f_{peak}$  and on the bandwidth of the signal. A general relationship between the peak frequency, the order of differentiation  $k$  and  $\alpha$ , can be derived by observing that the Fourier transform of the  $k$ -th derivative has the property:

$$X'_k(f) \propto f^k \cdot e^{-\frac{\pi \cdot f^2 \cdot \alpha^2}{2}} \quad (5)$$

which leads to the peak frequency of the  $k$ -th derivative  $f_{peak,k}$ :

$$f_{peak,k} = \sqrt{k} \frac{1}{\alpha \sqrt{\pi}} \quad (6)$$

The behaviour of the peak frequency as a function of  $\alpha$  for the set of first 15 derivatives is presented in Fig. 5.

As regards bandwidth, no analytical relation can be identified between differentiation order and -10 dB bandwidth. Numerical evaluation of the -10 dB bandwidth as a function of  $\alpha$  for the set of 15 derivatives shows that differentiation moves the energy on higher frequencies (Fig. 6). Fig. 7 shows the ESD of the first 15 derivatives.

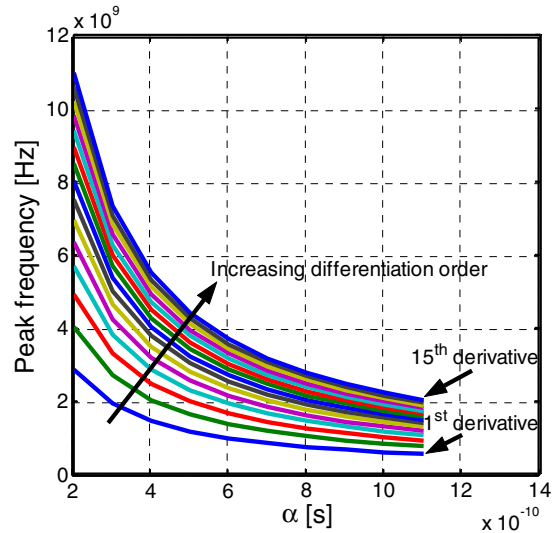


Figure 5. Effect of differentiation on peak frequency

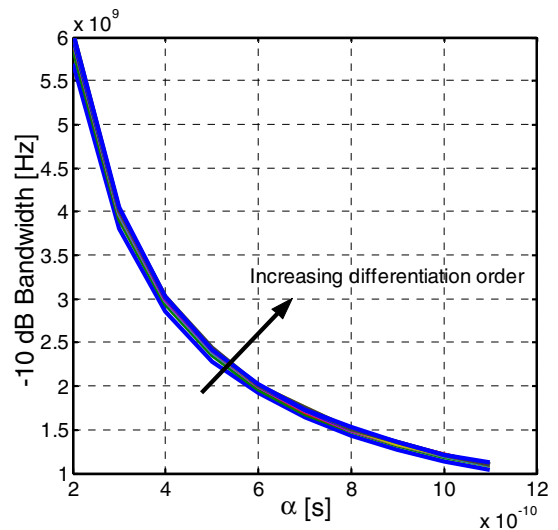


Figure 6. Effect of differentiation on -10 dB bandwidth

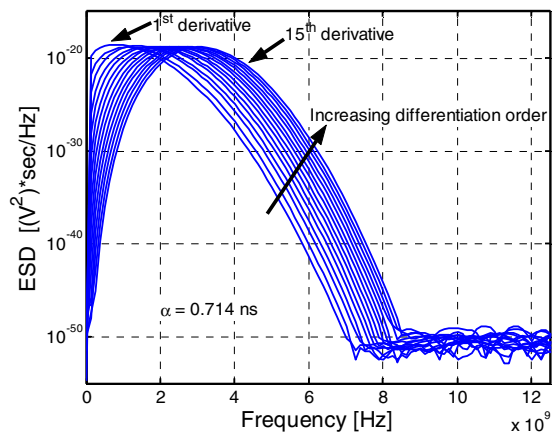


Figure 7. ESD of the first 15 derivatives of Gaussian pulse

#### IV. PULSE SHAPING AND EMISSION MASK APPROXIMATION

It was shown in Section III that both differentiation and pulse width variation affect the ESD of the transmitted waveform, and can be used to shape the PSD of the transmitted signal. In most cases however the flexibility in shaping the spectrum guaranteed by a single waveform is not sufficient to fulfil requirements. As an example, none of the derivatives of the Gaussian pulse leads to an efficient approximation of the FCC mask defined in Table 1. Higher flexibility may be achieved by using several base waveforms in order to produce the desired pulse shape. Linear combination of a set of independent functions is a straightforward manner for obtaining such a flexibility. A simple procedure for selecting the coefficients can be described as follows:

1. Choose a set of base functions BF;
2. Generate in a random way a set of coefficients, named S;
3. Check if the PSD of the signal based on the linear combination of the functions obtained with coefficients S satisfies the emission limits;
4. If yes, and this is the first set S verifying the mask, then initialize the procedure by setting  $S_B = S$ . If yes, and the procedure was already been initialized, then compare S with  $S_B$ : if S leads to a more efficient waveform than  $S_B$ , according to a well defined distance metrics, set  $S_B = S$ .
5. Repeat steps 2-4 until the distance between the mask and PSD of the generated signal falls below a threshold.

In particular, we consider the set of base functions BF be composed by the first 15 derivatives of the Gaussian pulse with  $\alpha=0.714$  ns. Fig. 8 shows the PSD of a signal obtained by linear combination of the above base functions plotted against the FCC emission mask, according to the procedure defined above.

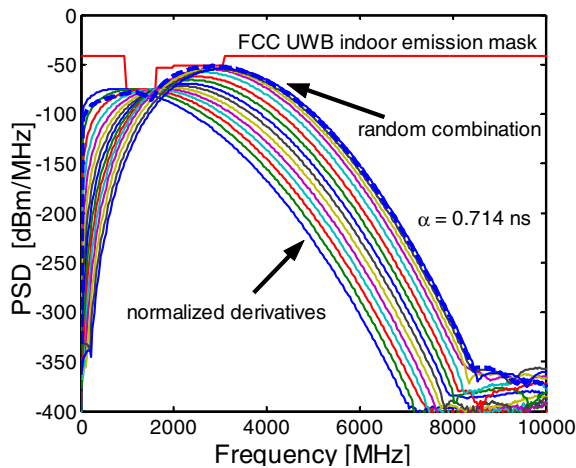


Figure 8. PSD of the base functions (solid plots) and of the combined waveform (dashed plot)

Fig. 8 shows that combination of several base functions leads to a better approximation of the emission mask. Furthermore the adoption of either a higher number of functions or functions with different  $\alpha$  values can lead

to better approximations of the mask, in particular at high frequencies.

Random selection of coefficients is obviously only one possibility in the choice of the linear combination: one can apply in a rather straightforward manner standard procedures for error minimization such as the Least-Square-Error (LSE). According to LSE, define the following error function:

$$e_s(t) = \int_{-\infty}^{+\infty} |e(t)|^2 dt = \int_{-\infty}^{+\infty} \left| f(t) - \sum_{k=0}^{N-1} a_k \cdot f_k(t) \right|^2 dt \quad (7)$$

and select the set of N coefficients  $a_1, \dots, a_N$  which minimizes  $e_s(t)$ .

Note that since requirements are defined in terms of PSD, the error must be defined between the reference PSD and the PSD of the linear combination of base functions. Therefore the following definition of error should be set:

$$E = \int_{-\infty}^{+\infty} |M(f) - F(f)|^2 df \quad (8)$$

where  $M(f)$  represents the emission mask, and  $F(f)$  the PSD of the linear combination. The problem can be solved equivalently by considering the corresponding autocorrelation functions  $R_M(f)$  and  $R_F(f)$  and minimizing the error:

$$E = \int_{-\infty}^{+\infty} |R_M(t) - R_F(t)|^2 df = \int_{-\infty}^{+\infty} \left| R_M(t) - \left[ \sum_{k=0}^{N-1} a_k^2 \int_{-\infty}^{+\infty} f_k(t) f_k^*(t + \tau) dt \right] \right|^2 df \quad (9)$$

Note however that such a criterion leads to a global minimized distance between reference and generated spectra, and may generally lead to a generated PSD which occasionally violates the mask, as shown in Fig. 9.

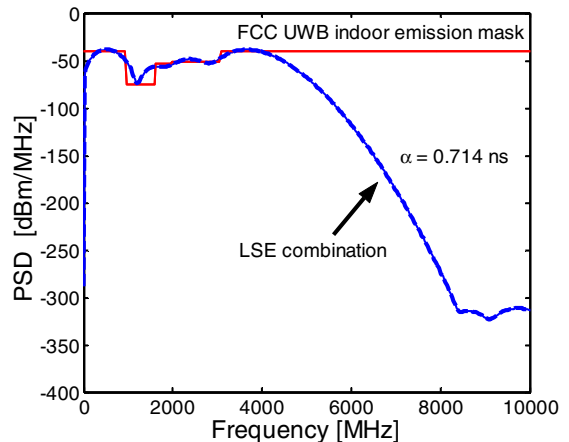


Figure 9. PSD of the linear combination of Gaussian waveforms versus FCC indoor emission mask

## V. PULSE SHAPING AND MUI MITIGATION

In UWB networks based on Time Hopping (TH)-CDMA MUI is the key factor in determining the maximum achievable bit rate. Pulse shaping can be a powerful tool in reducing the negative effect of MUI. A MAC protocol capable of selecting different pulse shapes, for example, can optimize network organization by assigning different waveforms to different groups of terminals, thus reducing the MUI noise suffered by each terminal and increasing network performance. This solution was tested in a scenario characterized by two disjoint UWB networks in the same physical area.

Each of the two networks, referred to as N1 and N2, was composed by 24 transmitting devices. The effect of MUI noise generated by interfering devices in both N1 and N2 on a useful link in N1 was analyzed by measuring the Bit Error Rate (BER) of the link as a function of the  $E_b/N_0$  ratio. The effect of pulse shaping was analyzed as follows. In all simulations the waveform considered at the output of the transmitting antennas in N1 was the second derivative of the Gaussian pulse, while the waveform adopted in N2 varied for each run of simulations. Results are shown in Fig. 10 for the cases of second, fourth and eighth derivative of the Gaussian pulse adopted as pulse waveform in N2.

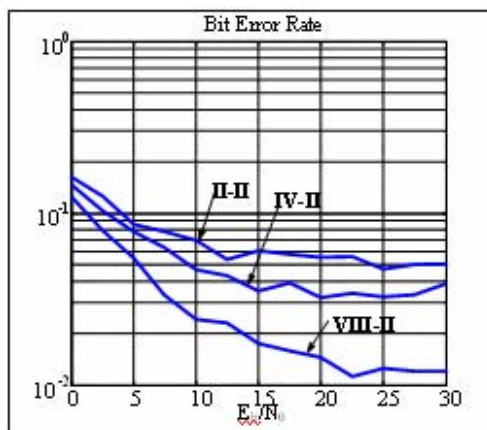


Figure 10. Bit Error Rate as a function of  $E_b/N_0$  for different waveform selection in overlapping UWB systems.

Results show that the adoption of different waveforms in the two networks reduces the BER, thanks to the different bandwidths occupied by the corresponding PSDs. This is also confirmed by the fact that higher derivatives adopted in N2 lead to lower BER, due to the effect of differentiation on bandwidth and peak frequency, shown in Section III. Note that the strategy of assigning different waveforms to different networks can be applied to linear combinations of base functions as well, guaranteeing at the same time low MUI interference and good approximation of the emission masks.

## VI. CONCLUSIONS

In this work a pulse shaping method for UWB transmissions was proposed. The method is based on the adoption of the Gaussian pulse as base function. The possibility of obtaining a set of base functions by means of pulse width variation and differentiation of the original

pulse was investigated, and the effect of these operations on the PSD of the resulting waveform was analyzed.

In particular, linear combination of a set of base functions obtained by differentiation of Gaussian pulse was proposed for approximating the power emission masks released by FCC, and two ways of determine the coefficients for such combination were proposed.

Results showed that better approximation of the mask can be achieved by adopting the linear combination in place of a single pulse. Mitigation of MUI between overlapping UWB networks by means of pulse shaping was finally analyzed. Simulations performed with two UWB networks and different waveform settings showed that the adoption of different waveforms in the two networks significantly reduces the Bit Error Rate, suggesting that pulse shaping capabilities should be included in optimal MAC design for UWB networks based on TH-CDMA.

## ACKNOWLEDGMENTS

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