Time Hopping Codes in Impulse Radio Multiple Access Communication Systems

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ABSTRACT

In this paper, we address the problem of designing time hopping codes for multiple access communication impulse radio systems. Time hopping techniques are first reviewed, and performance in terms of correlation properties and Power Spectral Density are given. A new family of time hopping codes is presented, and performance are derived. Finally, advantages and disadvantages of different time hopping codes in terms of resource assignment for multiple-access communication systems are analyzed and discussed.

INTRODUCTION

Impulse radio (IR) is a spread spectrum technique that uses very short duration pulses and pulse position modulation for transmitting information.
The transmitted signal is:

$$s(t) = \sum_{i=-\infty}^{\infty} \sum_{j=0}^{N_s-1} g\left(t - jT_f - b_i \boldsymbol{d}\right)$$

where $g(t)$ represents the pulse, N_s is the number of pulses per bit and $T_b = N_s \cdot T_f$ is the bit duration.

The sequence of b_k represents the information bits.

Multiple access capability is achieved using time hopping codes, and in this case the transmitted signal is:

$$s(t) = \sum_{i=-\infty}^{\infty} \sum_{j=0}^{N_s-1} g\left(t - jT_f - c_j T_c - b_i d\right)$$

An additional shift of $c_j T_c$ is provided by the hopping code, with $0 \le c_j \le N_h$ and $N_h \cdot T_c < T_f$.

When the number of users is N_u and the noise $n(t)$ is additive, the received signal is:

$$s_{rec}(t) = \sum_{k=1}^{N_u} \sum_{i=-\infty}^{\infty} \sum_{j=0}^{N_s-1} g\left(t - jT_f - c_j^{(k)} T_c - b_i^{(k)} \boldsymbol{d}\right) + n(t) \qquad (1)$$

Given that the receiver is the pulse correlator described in [1], the Signal to Noise Ratio SNR at the receiver output can be written as follows:

$$SNR = \frac{P_{av}}{N_s \boldsymbol{s}_a^2 (N_U - 1) + \boldsymbol{s}_{rec}^2} = \frac{\left(N_s m_p\right)^2}{N_s \boldsymbol{s}_a^2 (N_U - 1) + \boldsymbol{s}_{rec}^2} \qquad (2)$$

where P_{av} is the average power on the useful signal, $\boldsymbol{s}_{rec}^2$ is the power of the thermal noise, $\boldsymbol{s}_a^2$ is the interference average power provoked by one user in the time interval T_f, m_p is the signal at the correlator output in the same interval T_f.

The paper is organized as follows. In section 1 we review some time hopping techniques in terms of code construction, correlation properties and system performance. In the same section we introduce a new family of time hopping codes and their characteristics. In section 2 we present some results on the codes power spectral density; section 3 contains some considerations on the relationship between time hopping codes and the MAC layer. In section 4 the conclusions are given.

1. TIME HOPPING CODES: CODE CONSTRUCTION AND CORRELATION PROPERTIES

In time hopping spread-spectrum systems, multilevel sequences are used to specify which time interval T_c is used for transmission at any given time T_f.

As many users utilize the same resource (time), it is important to keep the mutual interference at a level as low as possible. This mutual interference is controlled by the cross correlation of the time hopping sequences.

Let $\boldsymbol{X} = \left(x_0, x_1, \ldots x_{N_p-1}\right)$ and $\boldsymbol{Y} = \left(y_0, y_1, \ldots y_{N_p-1}\right)$ denote two hopping sequences of period N_p. One of the best hopping sequence performance measures is provided by the periodic Hamming cross-correlation function, defined as:

$$H_{XY}(\boldsymbol{t}) = \sum_{i=0}^{N_p-1} h\left(x_i, y_{i+\boldsymbol{t}}\right) \quad 0 \le \boldsymbol{t} \le N_p - 1 \qquad (3)$$

where: $h(a,b) = \begin{cases} 0, & a \ne b \\ 1, & a = b \end{cases}$

$\boldsymbol{X}=(x_0,x_1,..x_{Np-1})$ and $\boldsymbol{Y}=(y_0,y_1,..y_{Np-1})$ denote two hopping sequences of period N_p; with x_i and $y_i \in \{0,1,\ldots N_h - 1\}$.

Equation (3) represents the number of coincidences or hits between the two sequences $\boldsymbol{X}$ and $\boldsymbol{Y}$.

In the analysis provided below all the sequences are supposed to be of the same length. Moreover, we are interested in codes correlation properties for $\boldsymbol{t} = 0$ (hypothesis of codeword synchronism) and for $\boldsymbol{t} \propto T_c,\ \boldsymbol{t} \le T_f - T_c$ (hypothesis of codeword asynchronism, but chip synchronism).

A. Pseudorandom code

The code is obtained as follows: for a given N_h, the N_p code symbols are obtained by causally extracting N_p numbers between 0 and $N_h - 1$. It is easy to see that the average number of hits between two codewords of length N_p is, in the synchronous and asynchronous case:

$$E\left(H_{XY}(0)\right) = E\left(H_{XY}(\boldsymbol{t})\right) = \frac{N_p}{N_h}$$

The signal to noise ratio given by (2) can therefore be specialized as follows:

$$SNR = \frac{\left(N_s m_p\right)^2}{N_s/N_h \cdot m_p^2 \cdot \left(N_U - 1\right) + s_{rec}^2}$$

B. Code construction 1

Given that $\boldsymbol{C}^k = \left(c_0^k, c_1^k, \ldots c_{N_p-1}^k\right)$ is the codeword of user k, the chips c_j^k are obtained as follows:

$c_j^k = \left[(k + j - 1) \bmod p\right]$ with p prime and $pT_c < T_f$. Index j=1..p identifies the time. This construction generates a family of p codes (p users) of length $N_p = p$ [3].

In the case of codeword synchronism, one hit occurs if and only if $c_j^{k_1} - c_j^{k_2} \equiv 0 \bmod p$, that is $\left(k_1 - k_2\right) \equiv 0 \bmod p$.

Therefore, if the codewords are synchronous, a perfect orthogonality is achieved. In this case, the SNR is:

$$SNR_{syn} = \frac{\left(N_s m_p\right)^2}{s_{rec}^2}$$

In the case of codeword asynchronism but chip synchronism, the condition of hit becomes:

$$\left[\left(k_1 + j\right) - \left(k_2 + i\right)\right] \equiv 0 \bmod p \qquad (4)$$

Equation (4) is satisfied if $i = \left(k_2 - k_1\right) \bmod p$.

Therefore, if the codewords are asynchronous, the perfect orthogonality is lost and one hit occurs between two different users.

The SNR is, in this case:

$$SNR_{asyn} = \frac{\left(N_s m_p\right)^2}{\left\lfloor N_s/N_p \right\rfloor \cdot m_p^2 \cdot \left(N_U - 1\right) + s_{rec}^2}$$

where $\lfloor x \rfloor$ represents the integer part of x.

C. Code construction 2: the new time hopping code

The time hop for a user is given by $c_j^{m,k} = \left(\frac{j}{k} + m\right) \bmod p$, p prime.

Index j=1..p-1 identifies the time; indexes k=1..p-1 and m=1..p-1 identify the user. In this way it is possible to generate a family of $(p-1)^2$ codes of length $N_p = p - 1$.

This construction was presented in [2] for generating $p - 1$ frequency hop signals carrying also the transmitted message m.

If the codewords are synchronous, the condition of hit is:

$$\left(\frac{j}{k_1} + m_1\right) - \left(\frac{j}{k_2} + m_2\right) \equiv 0 \bmod p \qquad (5)$$

As shown in [2], equation (5) has at most one solution and therefore two time hopping codewords $\boldsymbol{X}$ and $\boldsymbol{Y}$ can have at most one hit, that is $\max\left(H_{XY}(0)\right) = 1$.

It follows that the minimum SNR is:

$$SNR_{syn} = \frac{\left(N_s m_p\right)^2}{\left\lfloor N_s/N_p \right\rfloor \cdot m_p^2 \cdot \left(N_U - 1\right) + s_{rec}^2}$$

In the case of codeword asynchronism but chip synchronism, equation (5) becomes:

$$\left(\frac{j}{k_1} + m_1\right) - \left(\frac{i}{k_2} + m_2\right) \equiv 0 \bmod p \qquad (6)$$

It is possible to show [2] that equation (6) has at most two solutions and therefore two time hopping codewords can have at most two hits. Therefore, $\max\left(H_{XY}(\boldsymbol{t})\right) = 2$.

The minimum SNR is, in this case:

$$SNR_{asyn} = \frac{\left(N_s m_p\right)^2}{2 \cdot \left\lfloor N_s/N_p \right\rfloor \cdot m_p^2 \cdot \left(N_U - 1\right) + s_{rec}^2}$$

2. POWER SPECTRAL DENSITY

We consider the non-random signal:

$$s(t) = \sum_{n=-\infty}^{\infty} g\left(t - nT_f - c_n T_c\right) \qquad (7)$$

with $c_n \in \left\{0,1,\ldots N_h - 1\right\}$ and code length N_p.

The pulse $g(t)$ and its Power Spectral Density (PSD) are plotted in figures 1 (a) and 1 (b).

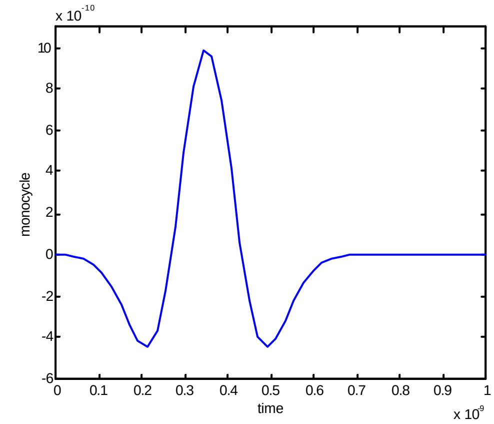

Fig. 1 (a): monocycle $g(t)$

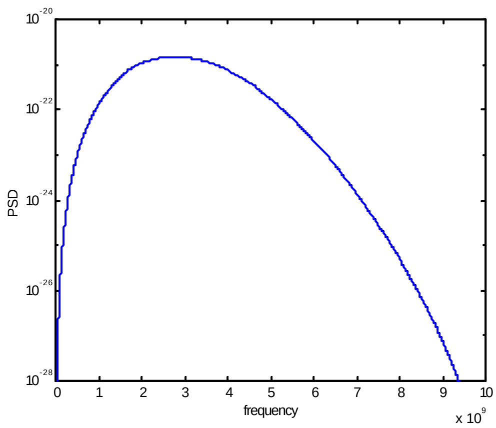

Fig. 1 (b): monocycle PSD.

In order to calculate the PSD of $s(t)$, we write (7) as follows:

$$s(t)=\sum_{k=-\infty}^{\infty}\left[\sum_{n=0}^{N_p-1} g\left(t-nT_f-c_nT_c-kT_p\right)\right]$$

with $T_p = N_p \cdot T_c$.

The Frequency Transform (FT) of $s(t)$ is:

$$S(f)=\sum_{k=-\infty}^{\infty}\exp\left(-j2\pi fkT_p\right)\left[\sum_{n=0}^{N_p-1} G(f)\exp\left(j2\pi f\left(-nT_f-c_nT\right)\right)\right] \tag{8}$$

with $G(f)=FT[g(t)]$.

As: $\sum_{k=-\infty}^{\infty}\exp\left(-j2\pi fkT_p\right)=\frac{1}{T_p}\sum_{k=-\infty}^{\infty}\delta\left(t-k/T_p\right)$,

(8) can be rewritten as follows:

$$S(f)=\frac{1}{T_p}\sum_{k=-\infty}^{\infty}\delta\left(f-k/T_p\right)\left[\sum_{n=0}^{N_p-1} G(f)\exp\left(-nT_f-c_nT_c\right)\right]$$

The Power Spectral Density of $s(t)$ is then:

$$PSD=\frac{1}{T_p^2}\sum_{k=-\infty}^{\infty}\delta\left(f-k/T_p\right)\left[\sum_{n=0}^{N_p-1} G(f)\exp\left(-nT_f-c_nT_c\right)\right]^2 \tag{9}$$

The signal PSD decreases when T_p increases and, meanwhile, the space between delta lines decreases.

The PSD shape is given by $|G(f)|^2$ (Fig. 1(b)).

Figures 2 and 3 report a plot of the power spectral densities of codes presented in section 2 in different situations; in such figures the delta lines are represented by dots. Figure 2 shows the power spectral density of a pseudo-random code with $N_p = N_h = 7$ (Fig. 2(a)) , a code from construction 1 with the same parameters (Fig. 2(b)) and a new code with $N_p = 6$ and $N_h = 7$. As expected, the power spectral density of different codes is comparable.

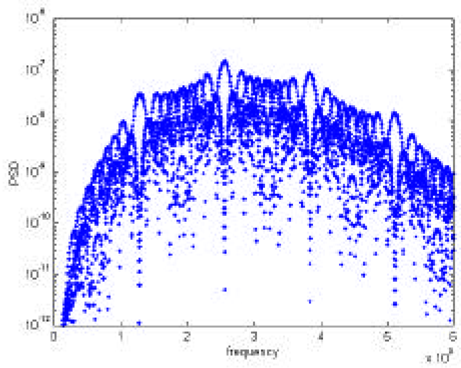

Fig.2 (a): PSD of a pseudo-random with $N_p = N_h = 7$;

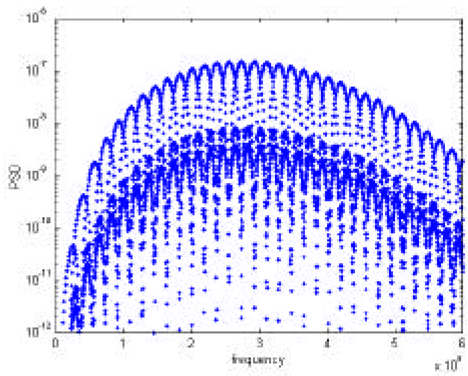

Fig. 2 (b): PSD of a code from construction 1 with $N_p = N_h = 7$;

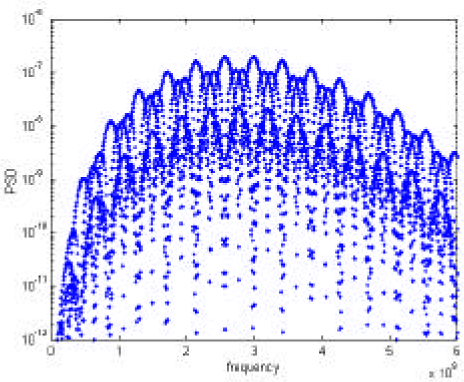

Fig.2 (c): PSD of a new code with N_p =6; N_h =7

The pseudo-random code shows some flexibility that the other codes do not have. For example, it is possible to augment the period N_p without changing N_h. Figure 3 shows the PSD of two pseudo-random codes with the same $N_h = 100$ and different N_p ($N_p = 7$ in figure 3(a) and $N_p = 128$ in figure 3(b)). Again, as it must be, the PSD with higher N_p has lines more dense and a lower maximum value.

3. TIME HOPPING CODES AND MAC LAYER: SOME CONSIDERATIONS

In this section we address some topics concerning the MAC layer and we discuss advantages and disadvantages of using the codes studied in the previous sections for multiple access.

The MAC layer has to fulfill QoS (including bit rate, delay, Packet Error Rate) requests from the network layer by changing some parameters (MAC parameters).

In a time hopping impulse radio system MAC parameters are the time hopping codes, the number of transmitted pulses per bit N_s, the nominal distance between two pulses T_f, the period of the time hopping code T_p, the pulse shape and its duration. All the MAC parameters are involved in the link performances.

Time hopping codes must have some important properties: they must be orthogonal or quasi-orthogonal, in order to generate small interference between users; they have to be addressed with a few parameters, in order to simplify the exchange of codes between communicating nodes; they must be numerous in the selected family; they must be defined for different lengths N_p and different N_h ($N_h \cdot T_c \le T_f$).

The pseudo-random code allows an unlimited number of users with code length N_p, but it is very difficult to address. In fact, it does not have a structure and the exchange of codes between two communicating nodes consists of the transmission of the whole code on a given control channel.

The other codes, since they have a construction, can be addressed with a few parameters, and in particular it is needed one integer (*m*) to address N_p users for code construction 1 and two integers (*k*,*m*) to address N_p^2 users for the new code. As a drawback, the new code looses, under the hypothesis of codeword asynchronism but chip synchronism, 3 dB compared to the other two codes. The above results are summarized in table 1.

4. CONCLUSIONS

In this paper we have studied the performance of three families of time hopping codes (pseudo-random code, code construction 1 and code construction2) in terms of cross-correlation properties, signal-to-noise ratio in a multiple access impulse radio system and power spectral density. The code from construction 2 is a new proposal for a time hopping code, even if it has been used for frequency hop signals carrying also the transmitted message [2].

It is shown that all the presented codes have good correlation properties in both the hypothesis of codeword and chip synchronism, keeping the mutual interference at a very low level, proportional to the ratio N_s/N_h. At the same time, it is possible to increase the SNR by increasing N_h and leaving unchanged all the other parameters.

Moreover, we have shown that a parameter which strongly affects the power spectral density is N_p. In particular, it is possible to decrease the PSD level of many decibels only augmenting the code period, without changing any other parameter. Finally, we considered and discussed the relationship between time hopping codes and the MAC layer in a multiple access impulse radio system.

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	Code Length	Number of allowed users	$\mathbf{SNR_{asyn}}$
Pseudo-random code	N_p	Unlimited	$SNR = \dfrac{(N_s m_p)^2}{\frac{N_s}{N_h} \cdot m_p^2 (N_U - 1) + s_{rec}^2}$
Code construction 1	$N_p = N_h$	N_p	$SNR = \dfrac{(N_s m_p)^2}{\frac{N_s}{N_p} \cdot m_p^2 (N_U - 1) + s_{rec}^2}$
Code construction 2	$N_p = N_h$	N_p^2	$SNR = \dfrac{(N_s m_p)^2}{2 \cdot \frac{N_s}{N_p} \cdot m_p^2 (N_U - 1) + s_{rec}^2}$

Table 1: code properties